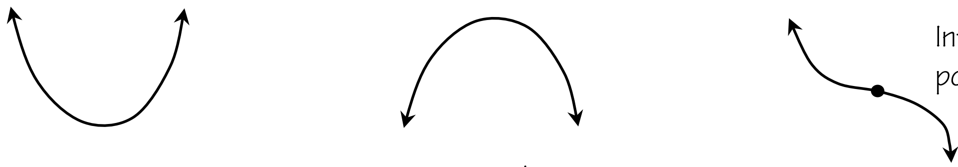
Concavity and the Second Derivative Test

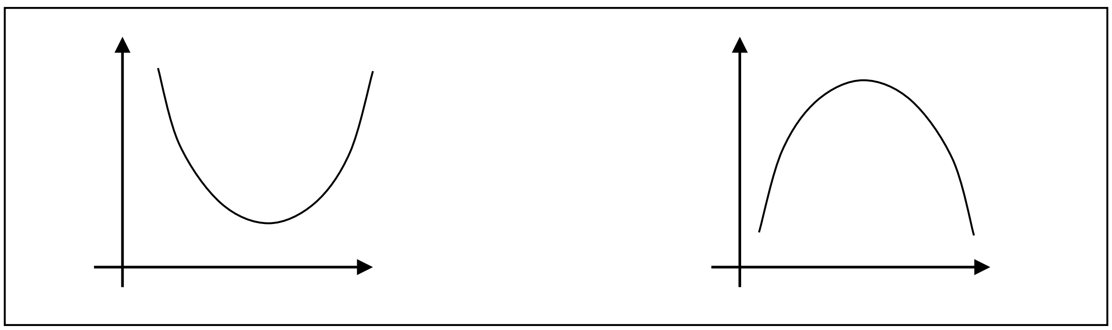
The first derivative describes the direction of the function. The second derivative describes the concavity of the original function. Concavity describes the direction of the curve, how it bends . . .



concave up   concave down

Just like direction, concavity of a curve can change, too. The points of change are called ***inflection points***.

CONCAVITY EXPLORATION: Draw small tangent lines at points along the curves below. What do you notice about the slopes of the tangent lines (the derivatives) as you move from left to right at these points?



TEST FOR CONCAVITY

If $f''(x) > 0$, then graph of f is concave up.

If $f''(x) < 0$, then graph of f is concave down.

SUMMARY OF FIRST AND SECOND DERIVATIVE TESTS RELATING FUNCTION BEHAVIOR:			
	$f'(x) > 0$	$f'(x) < 0$	$f'(x) = 0$
$f''(x) > 0$	Increasing and Concave Up	Decreasing and Concave Down	Relative Minimum and Concave Up
$f''(x) < 0$	Increasing and Concave Down	Decreasing and Concave Down	Relative Maximum and Concave Down
$f''(x) = 0$	Increasing and Inflection Point	Decreasing and Inflection Point	Function is "smooth, level" and a possible inflection point

 Jean Adams

EX #1: *Given* $f(x) = \frac{1}{3}x^3 - x$ *determine the open intervals on which the graph is concave upward or downward.*

STEP 1: Find the first derivative. $f'(x) =$

STEP2: Find the second derivative. $f''(x) =$

STEP 3: Find the critical values. $f''(x) = 0$

Make a sign chart for $f''(x)$

Test #

Critical #

Sign $f''(x)$

STEP 4: Find intervals for increasing/decreasing

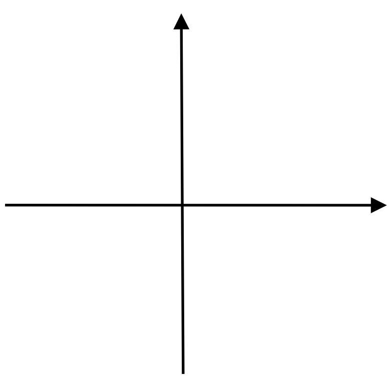
Increases:

Decreases:

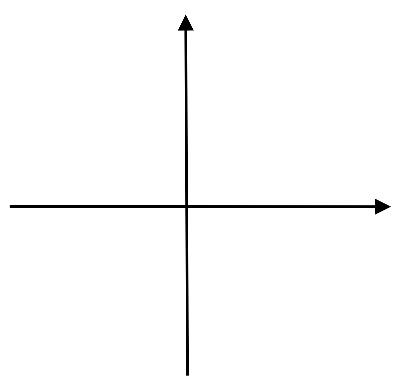
EX #2: *Graphs and Derivatives*

The concavity $(f''(x))$ and direction $(f'(x))$ of the function $(f(x))$ is related to the slope of the derivative.

$$f(x) = \frac{1}{3}x^3 - x$$



$$f'(x) = x^2 - 1$$



SUMMARY:

$f'(x)$ ____________________

$f''(x)$ ____________________

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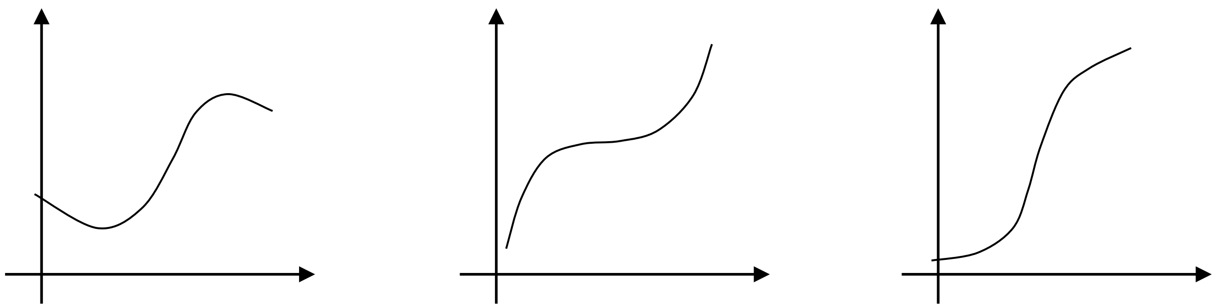
POINTS OF INFLECTION

The concavity of f changes at a point of inflection.

Where $f''(x) = 0$ or $f''(x)$ $does\ not\ exist.$



EX #3: Determine any points of inflection and discuss concavity of the graph of $f(x) = x^4 - 4x^3$

$f'(x) =$

Test #

Critical #

Sign $f''(x)$

$f''(x) =$

EX #4: Use the Second Derivative Test to determine the relative extrema for $f(x) = -3x^5 + 5x^3$

Step 1: Find critical numbers where $f'(x) = 0$

Step 2: Find $f''(x)$

Step 3: Find sign of $f''(x)$ for each critical number.

Point			
Sign of $f''(x)$			
Conclusion			

EX #5: Use First and Second Derivative Tests to determine behavior of f and graph.

Given: $f(x) = 3x^4 - 4x^3 + 6$

Procedure:

1. $f'(x) = 0$
2. Critical points
3. First Derivative Test
4. $f''(x) = 0$
5. Points of Inflection
6. Second Derivative Test
7. Summarize
8. Graph

$f'(x)$ Test

Conclusion:

$f''(x)$ Test

Conclusion: